Appendix C.

Kamel to Keplerian Transformation

MATHEMATICAL DESCRIPTION

OF THE TRANSFORMATION OF THE

GOES I-M KAMEL ORBITAL PARAMETERS

INTO KEPLERIAN ELEMENTS

Prepared for

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1. MATHEMATICAL BACKGROUND

The Kamel orbital parameters are a set of four parameters used in the GOES I/M Image Navigation and Registration (INR) system. These parameters depict the deviation of the true spacecraft orbit from the reference geostationary orbit.

KAMEL PARAMETER DEFINITIONS

The four Kamel parameters are defined as follows:

DR = The difference between the radial distance to the true instantaneous satellite position, R, and the nominal geostationary radial distance, R_0 (R_0 = 42164.365 km.):

$$DR = R - R_0 \quad (1)$$

Dlambda = The difference between the subsatellite longitude of the spacecraft orbit and the reference subsatellite longitude:

$$\text{Dlambda} = \text{ATAN2}[(yv_z - zv_y)/(xv_z - zv_x)] + \text{ATAN2}(L_s/PSI_s) - GHA - \text{Lambda}_0 \quad (2)$$

where (x,y,z) are the Geocentric Inertial Cartesian coordinates of the spacecraft position vector

(v_x,v_y,v_z) are the Geocentric Inertial Cartesian components of the spacecraft velocity vector

L_s and PSI_s are the Kamel parameters defined below

GHA is the Greenwich hour angle

Lambda_0 is the reference subsatellite longitude of the spacecraft (for example, Lambda_0 = -75 degrees East for GOES-East and Lambda_0 = -135 degrees East for GOES-West)

ATAN2 is an arctangent function that determines the proper quadrant of the resultant angle by evaluating the signs of the numerator and denominator terms of the argument.

Figure 1 shows the geometry of the orbit plane and the orbital angular momentum vector, $\vec{H}$, in a Cartesian coordinate system.

The magnitude of the orbital angular momentum is

$$H = sqrt\,[\,H_x^2 + H_y^2 + H_z^2]$$

where

$$H_x = yv_z - zv_y;\;\; H_y = zv_x - xv_z;\; H_z = xv_y - yv_x$$

In this Figure, $sqrt[H_x^2 + H_y^2]$ is the magnitude of the projection of the angular momentum vector onto the x-y plane and a is the angle that this projection makes with the x-axis. Then,

$$\cos(a) = H_x/sqrt[H_x^2 + H_y^2]$$

$$\sin(a) = -H_y/sqrt[H_x^2 + H_y^2]$$

However, a + RANODE = 90 degrees where RANODE is the right ascension of the ascending node. Consequently,

$$a = (90 - RANODE)$$

$$\cos(a) = \cos(90 - RANODE) = \sin(RANODE)$$

$$\sin(a) = \sin(90 - RANODE) = \cos(RANODE)$$

Then

$$\tan(RANODE) = H_x/(-H_y) = (yv_z - zv_y)/(xv_z - zv_x)$$

and

$$RANODE = ATAN2[(yv_z - zv_y)/(xv_z - zv_x)]$$

which is the first term of Dlambda. Thus, Dlambda is

$$Dlambda = RANODE + ATAN2(L_s/PSI_s) - GHA - Lambda_0 \qquad (3)$$

where $Lambda_0$ is the reference longitude contained in the GVAR documentation block.

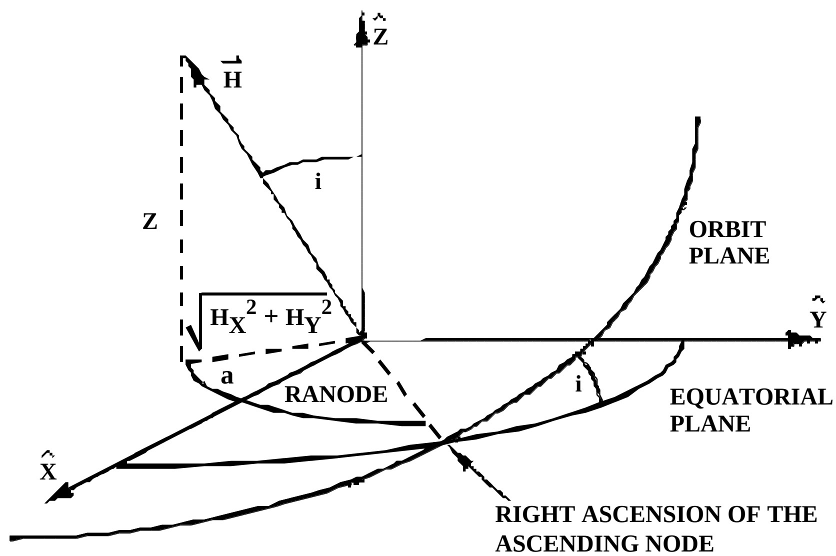

Figure 1. Orbital Angular Momentum Geometry

L_S = the sine of the geocentric latitude (the angle between the equatorial plane and the instantaneous latitude position):

$$L_S = z/R \quad (4)$$

PSI_S = the sine of the orbit yaw (the angle between the equatorial plane and the instantaneous velocity vector):

$$PSI_S = [v_Z R - z(xv_X + yv_y + zv_Z)/R]/H \quad (5)$$

The parameters L_S and PSI_S can also be expressed in terms of orbital angular variables. Figure 2 shows the geometry of an arc of the true spacecraft orbit relative to the equatorial plane. The arc u is along the orbital plane and is measured from the intersection of the orbital plane and equatorial plane (the ascending node) to the spacecraft position. In orbital mechanics, this angle, u, is called the argument of latitude. The arc S is along the equatorial plane and is measured from the ascending node to a point that is along the meridian containing the spacecraft position. The arc T is the angle measured along that meridian from the equatorial plane to the spacecraft position. This angle, T, is called the geocentric latitude. The arcs (u,S,T) form a spherical right triangle. From spherical trigonometry

$$\sin(T) = \sin(u)\sin(i)$$

where i, the inside angle formed by u and S, is the orbit inclination. Since T is the geocentric latitude, this equation defines the Kamel parameter L_S (the sine of the geocentric latitude) in terms of orbital angles:

$$L_S = \sin(u)\sin(i) \quad (6)$$

Also shown in Figure 2 is the angle a, or the orbit yaw, which is the angle that the instantaneous velocity vector makes with a parallel projection of the equatorial plane. From the geometry of Figure 2 it is evident that

$$a + b = 90 \text{ degrees}$$

Then from spherical trigonometry,

$$\cos(b) = \cos(90 - a) = \sin(a) = \cos(S)\sin(i)$$

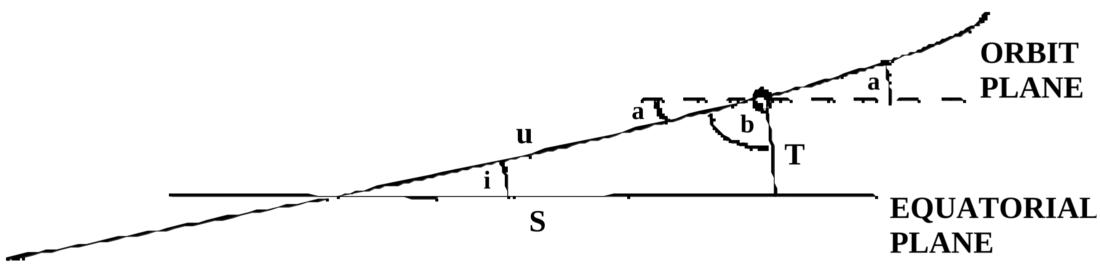

Figure 2. Relationship Between Orbit Geometry and the Angles of Geocentric Latitude and Orbit Yaw

For small inclination orbits, the angle S can be approximated to be the angle u with a high degree of accuracy. (For example, for an orbit with 0.5-degree inclination, this approximation is good to within 0.004 percent). Making this approximation,

$$\sin(a) = \cos(u)\sin(i)$$

Since a is the orbit yaw, this equation defines the Kamel parameter PSI_s (the sine of the orbit yaw) in terms of orbital angles:

$$PSI_S = \cos(u)\sin(i) \qquad (7)$$

Dividing equation (7) by equation (6) gives

$$\tan(u) = L_S/PSI_S$$

or

$$u = ATAN2(L_S/PSI_S)$$

which is the second term in equation (3) for Dlambda. Substituting the above expression for u into equation (3) gives an expression for Dlambda completely in terms of orbital angles:

$$Dlambda = RANODE + u - GHA - Lambda_0 \qquad (8)$$

Note that a singularity exists as PSI_S (or orbit yaw) goes to zero. In the event that this occurs, then u is determined from L_S in the following manner:

- If $L_s > 0$ then u = 90 degrees.
- If $L_s < 0$ then u = 270 degrees.
- If $L_S = PSI_S = 0$ then the inclination and right ascension of the ascending node also go to zero and

$$u = Dlambda + GHA + Lambda_0.$$

EXPRESSION OF KAMEL PARAMETERS IN TERMS OF IMC ORBIT COEFFICIENTS

The four Kamel parameters are obtained from the 42 IMC orbit coefficients in documentation block of the GVAR data for the Imager and Sounder. Table 1 lists the word locations in the respective documentation blocks for these IMC coefficients. In terms of these coefficients, the Kamel parameters are the following:

$$\begin{aligned} DR = {} & A14 + A15\cos(w_0t) + A16\sin(w_0t) + A17\cos(2w_0t) \\ & + A18\sin(2w_0t) + A19\cos(w_1t) + A20\sin(w_1t) \\ & + A21\cos(w_2t) + A22\sin(w_2t) \\ & + w_0t \{ A23\cos(w_0t) + A24\sin(w_0t) \} \end{aligned} \quad (9)$$

$$\begin{aligned} Dlambda = {} & A1 + A2w_0t + A3w_0^2t^2 \\ & + 2[A4\sin(w_0t) + A5\cos(w_0t) + A6\sin(2w_0t) \\ & + A8\sin(w_1t) + A9\cos(w_1t) + A10\sin(w_2t) \\ & + A7\cos(2w_0t) + A11\cos(w_2t)] \\ & + 2w_0t[A12\sin(w_0t) + A13\cos(w_0t)] \end{aligned} \quad (10)$$

$$\begin{aligned} L_s = {} & A25 + A26\cos(w_0t) + A27\sin(w_0t) + A28\cos(2w_0t) \\ & + A29\sin(2w_0t) + w_0t[A30\cos(w_0t) + A31\sin(w_0t)] \\ & + A32\cos(w_2t) + A33\sin(w_2t) \end{aligned} \quad (11)$$

$$\begin{aligned} PSI_s = {} & A34 + A35\sin(w_0t) + A36\cos(w_0t) \\ & + A37\sin(2w_0t) + A38\cos(2w_0t) \\ & + w_0t[A39\sin(w_0t) + A40\cos(w_0t)] \\ & + A41\sin(w_2t) + A42\cos(w_2t) \end{aligned} \quad (12)$$

where w_0 = 0.7292115D-4 radians/second
= the sidereal rotation rate of the Earth

$2*w_0$ = 1.458423D-4 radians/second
= the second harmonic of the Earth's rotation rate

Table 1. Word Locations of the 42 IMC Orbit Coefficients in the Imager and Sounder Documentation Block 0

Coefficient	Description (Units)	Imager Documentation Block 0 Word Location	Sounder Documentation Block 0 Word Location
A1	Change in longitude from reference (radians)	347	375
A2	Change in longitude from reference (radians)	351	379
A3	Change in longitude from reference (radians)	355	383
A4	Change in longitude from reference (radians)	359	387
A5	Change in longitude from reference (radians)	363	391
A6	Change in longitude from reference (radians)	367	395
A7	Change in longitude from reference (radians)	371	399
A8	Change in longitude from reference (radians)	375	403
A9	Change in longitude from reference (radians)	379	407
A10	Change in longitude from reference (radians)	383	411
A11	Change in longitude from reference (radians)	387	415
A12	Change in longitude from reference (radians)	391	419
A13	Change in longitude from reference (radians)	395	423
A14	Change in radial distance from reference (km)	399	427
A15	Change in radial distance from reference (km)	403	431
A16	Change in radial distance from reference (km)	407	435
A17	Change in radial distance from reference (km)	411	439
A18	Change in radial distance from reference (km)	415	443
A19	Change in radial distance from reference (km)	419	447
A20	Change in radial distance from reference (km)	423	451
A21	Change in radial distance from reference (km)	427	455
A22	Change in radial distance from reference (km)	431	459
A23	Change in radial distance from reference (km)	435	463
A24	Change in radial distance from reference (km)	439	467
A25	Sine of geocentric latitude (no units)	443	471
A26	Sine of geocentric latitude (no units)	447	475
A27	Sine of geocentric latitude (no units)	451	479
A28	Sine of geocentric latitude (no units)	455	483
A29	Sine of geocentric latitude (no units)	459	487
A30	Sine of geocentric latitude (no units)	463	491
A31	Sine of geocentric latitude (no units)	467	495
A32	Sine of geocentric latitude (no units)	471	499
A33	Sine of geocentric latitude (no units)	475	503
A34	Sine of orbit yaw (no units)	479	507
A35	Sine of orbit yaw (no units)	483	511
A36	Sine of orbit yaw (no units)	487	515
A37	Sine of orbit yaw (no units)	491	519
A38	Sine of orbit yaw (no units)	495	523
A39	Sine of orbit yaw (no units)	499	527
A40	Sine of orbit yaw (no units)	503	531
A41	Sine of orbit yaw (no units)	507	535
A42	Sine of orbit yaw (no units)	511	539

w_1 = 0.1405004D-3 radians/second

= $2*w_0 - 2*w_m$

= the frequency of twice the mean spacecraft orbital motion relative to the mean lunar motion, w_m

w_2 = 0.6759791D-4 radians/second

= $w_0 - 2*w_m$

= the frequency of the mean spacecraft orbital motion relative to the mean lunar motion

w_m = 0.0533219D-4 radians/second

= the mean lunar motion defined as the rate of change of the mean longitude of the moon, measured along the lunar orbit from the mean equinox of date of the ecliptic plane to the ascending node of the lunar orbit, then along the lunar orbit

t = the time elapsed since epoch in seconds

DERIVATION OF THE SATELLITE POSITION VECTOR

The equations (1), (6), (7), and (8) can be used to find the spacecraft position vector in the following manner. Figure 3 shows two coordinate systems—the Cartesian coordinate system (x,y,z) and the Coordinate system (U,V,W) where:

U is in the direction pointing to the spacecraft position

W is in the direction of orbit normal

V completes the right-handed coordinate system

A vector in the Cartesian coordinate system can be rotated into the (UVW)-system through the following three rotations:

$$\begin{matrix} \cos(RANODE) & \sin(RANODE) & 0 \\ -\sin(RANODE) & \cos(RANODE) & 0 \\ 0 & 0 & 1 \end{matrix} \qquad (13)$$

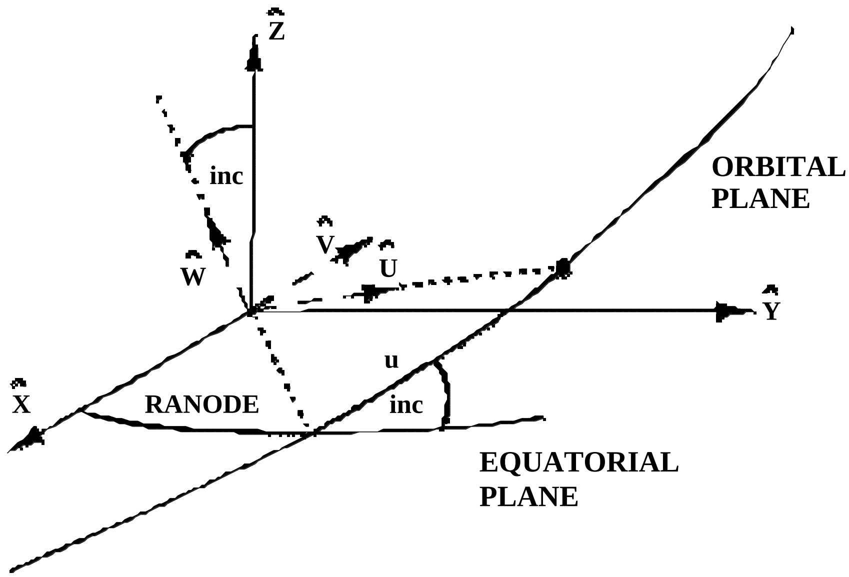

Figure 3. Geometry of Rotation from Inertial Coordinates to Orbital Coordinates

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & \sin(i) \\ 0 & -\sin(i) & \cos(i) \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} \cos(u) & \sin(u) & 0 \\ -\sin(u) & \cos(u) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{15}$$

The full rotation is a product of these three rotations:

$$T = \begin{pmatrix} (\cos(u)\cos(RANODE) - \sin(u)\sin(RANODE)\cos(i)) & (\cos(u)\sin(RANODE) + \sin(u)\cos(RANODE)\cos(i)) & \sin(u)\sin(i) \\ (-\sin(u)\cos(RANODE) - \cos(u)\sin(RANODE)\cos(i)) & (-\sin(u)\sin(RANODE) + \cos(u)\cos(RANODE)\cos(i)) & \cos(u)\sin(i) \\ \sin(RANODE)\sin(i) & -\sin(i)\cos(RANODE) & \cos(i) \end{pmatrix}$$

The (xyz)-system is then rotated into the (UVW)-system through

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} U \\ V \\ W \end{pmatrix} \tag{16}$$

Using this rotation, the Cartesian components of the unit vector U, which is the unit vector pointing to the spacecraft position vector, are the following:

$$U_x = \cos(u)\cos(RANODE) - \sin(u)\sin(RANODE)\cos(i) \quad (17)$$

$$U_y = \cos(u)\sin(RANODE) + \sin(u)\cos(RANODE)\cos(i) \quad (18)$$

$$U_Z = \sin(u)\sin(i) \quad (19)$$

The angles u, RANODE, and i can be obtained from the Kamel parameter equations (6), (7), and (8):

$$u = ATAN2(L_s/PSI_s) \quad (20)$$

where

if $PSI_s = 0$ and $L_s > 0$ then u = 90 degrees;

if $PSI_s = 0$ and $L_s < 0$ then u = 270 degrees;

if $L_s = PSI_s = 0$ then $u = Dlambda + GHA + Lambda_0$

$$RANODE = Dlambda - u + GHA + Lambda_0 \quad (21)$$

$$i = \arcsin[sqrt(L_s^2 + PSI_s^2)] \quad (22)$$

The Kamel parameter equation (1) can then be used to obtain the spacecraft position vector. From equation (1), the magnitude of the spacecraft position vector is

$$R = R_0 + DR \quad (23)$$

Then the (x,y,z) Cartesian components of the spacecraft position are:

$$x = RU_x \quad (24)$$

$$y = RU_y \quad (25)$$

$$z = RU_z \quad (26)$$

DERIVATION OF THE SATELLITE VELOCITY VECTOR

The velocity is derived using the partial derivatives of the position vector given in equations (24) through (26) as follows:

$$v_x = dx/dt = (dR/dt)U_x + R(dU_x/dt) \tag{27}$$

$$v_y = dy/dt = (dR/dt)U_y + R(dU_y/dt) \tag{28}$$

$$v_z = dz/dt = (dR/dt)U_z + R(dU_z/dt) \tag{29}$$

Using equations (17) through (23) the following expressions are obtained for the quantities dR/dt and dU_i/dt (i = x,y,z) in terms of the four Kamel parameters and their time derivatives:

$$dR/dt = d(DR)/dt \tag{30}$$

$$\begin{aligned} dU_x/dt = & - (du/dt)\sin(u)\cos(RANODE) \\ & - (dRANODE/dt)\cos(u)\sin(RANODE) \\ & - (du/dt)\cos(u)\sin(RANODE)\cos(i) \\ & - (dRANODE/dt)\sin(u)\cos(RANODE)\cos(i) \\ & + (di/dt)\sin(u)\sin(RANODE)\sin(i) \end{aligned} \tag{31}$$

$$\begin{aligned} dU_y/dt = & - (du/dt)\sin(u)\sin(RANODE) \\ & + (dRANODE/dt)\cos(u)\cos(RANODE) \\ & + (du/dt)\cos(u)\cos(RANODE)\cos(i) \\ & - (dRANODE/dt)\sin(u)\sin(RANODE)\cos(i) \\ & - (di/dt)\sin(u)\cos(RANODE)\sin(i) \end{aligned} \tag{32}$$

$$dU_z/dt = dL_s/dt \tag{33}$$

where

$$du/dt = [(dL_s/dt)PSI_s - (dPSI_s/dt)L_s]/\sin^2(i) \quad (34)$$

$$dRANODE/dt = dDlambda/dt - du/dt + dGHA/dt \quad (35)$$

(dGHA/dt is the sidereal rate of the Earth's rotation
$= w_0 = 0.7292115D\text{-}4$ radians/second)

$$di/dt = [L_s(dL_s/dt) + PSI_s(dPSI_s/dt)]/(\sin(i)\cos(i)) \quad (36)$$

Equations (34) and (36) encounter singularities when the inclination goes to zero. These singularities are avoided, however, when these equations, along with equation (35), are directly substituted into equations (31) and (32). Using the following three trigonometric relations

$$(1 - \cos i)/\sin^2 i = 1/[2\cos^2(i/2)]$$

$$\sin(RANODE)\cos(u) - \cos(RANODE)\sin(u) = \sin(RANODE - u)$$

$$\sin(RANODE)\sin(u) + \cos(RANODE)\cos(u) = \cos(RANODE - u)$$

the expressions for dU_x/dt (equation (31)) and dU_y/dt (equation (32)) reduce to forms that are more appropriate for small inclination orbits:

$$\begin{aligned} dU_x/dt = {} & ((dL_s/dt)PSI_s - (dPSI_s/dt)L_s)\sin(RANODE - u)/(2\cos^2(i/2)) \\ & + ((dL_s/dt)L_s + (dPSI_s/dt)PSI_s)(\sin(u)\sin(RANODE)/\cos(i)) \\ & - dDlambda/dt(\cos(u)\sin(RANODE) + \sin(u)\cos(RANODE)\cos(i)) \quad (37) \\ & - dGHA/dt[\cos(u)\sin(RANODE) + \sin(u)\cos(RANODE)\cos(i)] \end{aligned}$$

$$\begin{aligned} dU_y/dt = {} & ((dPSI_s/dt)L_s - (dL_s/dt)PSI_s)\cos(RANODE - u)/ [2\cos^2(i/2)] \\ & - ((dL_s/dt)L_s + (dPSI_s/dt)PSI_s)\sin(u)\cos(RANODE)/\cos(i) \\ & + dDlambda/dt(\cos(u)\cos(RANODE) - \sin(u)\sin(RANODE)\cos(i)) \quad (38) \\ & + dGHA/dt[\cos(u)\cos(RANODE) - \sin(u)\sin(RANODE)\cos(i)] \end{aligned}$$

Using equations (9) through (12) the time derivatives of the Kamel parameters are the following:

$$
\begin{aligned}
dDR/dt = \; & w_0[\, -Al5\sin(w_0 t) + Al6\cos(w_0 t)\,] \\
& + 2w_0[\, -Al7\sin(2w_0 t) + Al8\cos(2w_0 t)\,] \\
& + w_1[\, -A19\sin(w_1 t) + A20\cos(w_1 t)\,] \\
& + w_2[\, -A21\sin(w_2 t) + A22\cos(w_2 t)\,] \\
& + w_0[\, A23\cos(w_0 t) + A24\sin(w_0 t)\,] \\
& + w_0^2 t[\, -A23\sin(w_0 t) + A24\cos(w_0 t)\,]
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
dDlambda/dt = \; & A2w_0 + 2A3w_0^2 t + 2w_0[\, A4\cos(w_0 t) - A5\sin(w_0 t)\,] \\
& + 4w_0[\, A6\cos(2w_0 t) - A7\sin(2w_0 t)\,] \\
& + 2w_1[\, A8\cos(w_1 t) - A9\sin(w_1 t)\,] \\
& + 2w_2[\, A10\cos(w_2 t) - A11\sin(w_2 t)\,] \\
& + 2w_0[\, A12\sin(w_0 t) + A13\cos(w_0 t)\,] \\
& + 2w_0^2 t[\, A12\cos(w_0 t) - A13\sin(w_0 t)\,]
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
dL_s/dt = \; & w_0[\, -A26\sin(w_0 t) + A27\cos(w_0 t)] \\
& + 2w_0[\, -A28\sin(2w_0 t) + A29\cos(2w_0 t)] \\
& + w_0[\, A30\cos(w_0 t) + A31\sin(w_0 t)] \\
& + w_0^2 t[\, -A30\sin(w_0 t) + A31\cos(w_0 t)] \\
& + w_2[\, -A32\sin(w_2 t) + A33\cos(w_2 t)\,]
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
dPSI_s/dt = \; & w_0[A35\cos(w_0 t) - A36\sin(w_0 t)] \\
& + 2w_0[\, A37\cos(2w_0 t) - A38\sin(2w_0 t)] \\
& + w_0[\, A39\sin(w_0 t) + A40\cos(w_0 t)] \\
& + w_0^2 t[\, A39\cos(w_0 t) - A40\sin(w_0 t)] \\
& + w_2[\, A41\cos(w_2 t) - A42\sin(w_2 t)]
\end{aligned} \tag{42}
$$

The velocity vector components (equations (27) through (29)) are then obtained with substitutions of equations (39) through (42) into equations (30), (33), (37), and (38) followed by substitution of equations (30), (33), (37), and (38) into equations (27) through (29).

CARTESIAN ELEMENT CONVERSION TO KEPLERIAN ELEMENTS

The Keplerian elements can now be obtained from the position and velocity vectors through the following:

Semi-major axis (sma):

$$sma = R/[2 - (RV^2)/mu] \tag{43}$$

where R = the magnitude of the position vector
V = the magnitude of the velocity vector
mu = the Earth's gravitational constant

Eccentricity (ecc):

$$ecc = sqrt[1 - (p/sma)] \tag{44}$$

where $p = [(RV)^2 - (\vec{R}\ \ \vec{V})^2]/mu$

$\vec{R}\ \ \vec{V} = xv_x + yv_y + zv_z$

Inclination (inc):

$$inc = arcsin [sqrt(L_s^2 + PSI_s^2)] \tag{45}$$

Right ascension of the ascending node (RANODE):

$$RANODE = Dlambda - u + GHA + Lambda_0 \tag{46}$$

True anomaly (TA):

$$TA = ATAN2 [\{(\vec{R}\ \ \vec{V})sqrt(p/mu)\} /(p - R)] \tag{47a}$$

If (p - R) is zero (or, equivalently, if the eccentricity is zero) then the argument of perigee is undefined. (Note: When the eccentricity is near zero the argument of perigee is ill-defined due to limitations in mathematical algorithms and floating point computations.) In that case, the argument of perigee is set to zero and the true anomaly is

$$TA = u \tag{47b}$$

Argument of perigee (ARGPER):

If the eccentricity is zero then

ARGPER = 0 (48a)

else

ARGPER = u - TA (48b)

Eccentric anomaly (EA):

arg = sqrt[(1 - ecc)/(1 + ecc)]
EA = 2*arctangent[arg*tangent (TA/2)] (49)

Mean anomaly (MA):

MA = EA - ecc*sin(EA) (50)

2. GENERAL ALGORITHM

The following algorithm can be used as the basis for a computer program that will obtain the Keplerian elements from the 42 IMC coefficients contained in the imager and sounder documentation blocks (see Table 1). It is organized in four steps, each of which can be programmed as subroutines if desired. These steps are the following:

1. Compute the four Kamel parameters and their time derivatives from the 42 IMC coefficients.

2. Compute three orbit angles, their time derivatives, and the magnitude of the spacecraft position from the four Kamel parameters and their time derivatives.

3. Compute the position and velocity vectors from three orbit angles, their time derivatives, and the magnitude of the spacecraft position.

4. Compute the Keplerian elements from the position and velocity vectors.

The algorithm is written in pseudocode form with variable names given in FORTRAN like form. Comments that are not part of the algorithm are contained within double parentheses.

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((  ALGORITHM TO CONVERT THE KAMEL PARAMETERS TO                          ))
((  KEPLERIAN ELEMENTS.                                                   ))
((                                                                        ))
((           NOTES:                                                       ))
((                                                                        ))
((                                                                        ))
((           1.  THIS ALGORITHM SHOULD BE PROGRAMMED IN DOUBLE            ))
((               PRECISION.                                               ))
((                                                                        ))
((           2.  INTRINSIC FUNCTIONS (SUCH AS SINE, COSINE, ETC.)         ))
((               ARE LISTED IN LOWER CASE IN THIS ALGORITHM.              ))
((                                                                        ))
((                                                                        ))
((           INPUT PARAMETERS:                                            ))
((                                                                        ))
((           A(42) - A 1 X 42 ARRAY CONTAINING THE 42 IMC ORBIT           ))
((           COEFFICIENTS                                                 ))

((                                                                        ))
((           T - TIME IN SECONDS SINCE EPOCH                              ))
((                                                                        ))
((                                                                        ))
((           GHA - THE GREENWICH HOUR ANGLE (IN RADIANS) AT THE           ))
((           TIME OF THE ABOVE COEFFICIENTS                               ))
((                                                                        ))
```

```
((          LAM0 - THE REFERENCE SUBSATELLITE LONGITUDE FOR          ))
((          THE SPACECRAFT IN RADIANS (FOR EXAMPLE, -1.308997        ))
((          RADIANS (-75 DEGREES EAST) FOR GOES-EAST OR              ))
((          -2.356194 RADIANS (-135 DEGREES EAST) FOR                ))
((          GOES-WEST)                                               ))
((                                                                   ))
((                                                                   ))
((         CONSTANT PARAMETERS:                                      ))
((                                                                   ))
((             W0  -  (THE FUNDAMENTAL FREQUENCY IN THE IMC          ))
((                    EXPANSION) = 0.7292115D-4 RADIANS/SECOND       ))
((                                                                   ))
((                                                                   ))
((             W1  -  (THE FREQUENCY OF TWICE THE MEAN               ))
((                    SPACECRAFT ORBITAL MOTION RELATIVE TO THE      ))
((                    MEAN LUNAR MOTION) = 0.1405004D-3 RADIANS/     ))
((                    SECOND                                         ))
((                                                                   ))
((                                                                   ))
((             W2  -  (THE FREQUENCY OF THE MEAN SPACECRAFT          ))
((                    ORBITAL MOTION RELATIVE TO THE MEAN LUNAR      ))
((                    MOTION) = 0.6759791D-4 RADIANS/SECOND          ))
((                                                                   ))
((                                                                   ))
((             MU -   (THE EARTH'S GRAVITATIONAL CONSTANT)           ))
((                    = 3.9860044D5 KM**3/SECOND**2                  ))
((                                                                   ))
((                                                                   ))
((             R0 -   (THE NOMINAL GEOSTATIONARY ORBIT RADIAL        ))
((                    DISTANCE) = 42164.365D0 KM                     ))
((                                                                   ))
((                                                                   ))
((             GHADT -   (THE RATE OF CHANGE OF THE GREENWICH        ))
((                       HOUR ANGLE) = 0.7292115D-4 RADIANS/SECOND   ))
((                                                                   ))
((                                                                   ))
((          BEGIN COMPUTATION:                                       ))
((                                                                   ))
((          1.  COMPUTE THE FOUR KAMEL PARAMETERS AND THEIR          ))
((              TIME DERIVATIVES FROM THE 42 IMC COEFFICIENTS:       ))
((                                                                   ))
((              1.1   COMPUTE INTERMEDIATE SINE AND COSINE VALUES:   ))
((                                                                   ))
```

C0 = cosine(W0*T)

S0 = sine(W0*T)

C20 = cosine(2.0D0*W0*T)

S20 = sine(2.0D0*W0*T)

```
Cl = cosine(W1*T)

S1 = sine(W1*T)

C2 = cosine(W2*T)

S2 = sine(W2*T)
```

(( ))
((1.2 COMPUTE DR, THE RADIAL DISTANCE KAMEL PARAMETER:))
(( ))

```
DR =  A(14) + A(l5)*C0 + A(16)*S0 + A(17)*C20 + A(18)*S20
      + A(19)*C1 + A(20)*S1 + A(21)*C2 + A(22)*S2
      + W0*T*(A(23)*C0 + A(24)*S0)
```

(( ))
((1.3 COMPUTE DRDT, THE TIME DERIVATIVE OF THE RADIAL DISTANCE KAMEL PARAMETER:))
(( ))

```
DRDT =W0*(-A(15)*S0 + A(16)*C0) + 2.0D0*W0*(-A(17)*S20
        + A(18)*C20) + W1*(-A(19)*S1 + A(20)*C1)
        + W2*(-A(21)*S2 + A(22)*C2) + W0*(A(23)*C0
        + A(24)*S0) + (W0**2)*T*(-A(23)*S0 + A(24)*C0)
```

(( ))
((1.4 COMPUTE DLAM, THE LONGITUDINAL KAMEL PARAMETER:))
(( ))

```
DLAM =A(1) + A(2)*W0*T + A(3)*((W0*T)**2)
        + 2.0D0*(A(4)*S0 + A(5)*C0 + A(6)*S20 + A(7)*C20
        + A(8)*S1 + A(9)*C1 + A(10)*S2 + A(11)*C2)
        + 2.0D0*W0*T*(A(12)*S0 + A(13)*C0)
```

(( ))

```
((                1.5  COMPUTE DLAMDT, THE TIME DERIVATIVE OF THE          ))
((                     LONGITUDINAL KAMEL PARAMETER:                       ))
((                                                                         ))

                       DLAMDT = A(2)*W0 + 2.0D0*A(3)*(W0**2)*T
                                + 2.0D0*W0*(A(4)*C0 - A(5)*S0)
                                + 4.0D0*W0*(A(6)*C20 - A(7)*S20)
                                + 2.0D0*(W1*(A(8)*C1 - A(9)*S1) + W2*(A(10)*C2
                                - A(11)*S2)) + 2.0D0*W0*(A(12)*S0 + A(13)*C0)
                                + 2.0D0*(W0**2)*T*(A(12)*C0 - A(13)*S0)

((                                                                         ))
((                1.6  COMPUTE LS, THE LATITUDINAL KAMEL PARAMETER:        ))
((                                                                         ))

                       LS =     A(25) + A(26)*C0 + A(27)*S0 + A(28)*C20 + A(29)*S20
                                + W0*T*(A(30)*C0 + A(31)*S0) + A(32)*C2 + A(33)*S2

((                                                                         ))
((                1.7  COMPUTE LSDT, THE TIME DERIVATIVE OF THE            ))
((                     LATITUDINAL KAMEL PARAMETER:                        ))
((                                                                         ))

                       LSDT = W0*(-A(26)*S0 + A(27)*C0) + 2.0D0*W0*(-A(28)*S20
                              + A(29)*C20) + W0*(A(30)*C0 + A(31)*S0
                              + (W0**2)*T*(-A(30)*S0 + A(31 )*C0)
                              + W2*(-A(32)*S2 + A(33)*C2)

((                                                                         ))
((                1.8  COMPUTE PSIS, THE ORBIT YAW KAMEL PARAMETER:        ))
((                                                                         ))

                       PSIS = A(34) + A(35)*S0 + A(36)*C0 + A(37)*S20
                            + A(38)*C20 + W0*T*(A(39)*S0 + A(40)*C0)
                            + A(41)*S2 + A(42)*C2
```

(( ))
((1.9 COMPUTE PSISDT, THE TIME DERIVATIVE OF THE))
((ORBIT YAW KAMEL PARAMETER:))
(( ))

PSISDT = W0*(A(35)*C0 - A(36)*S0) + 2.0D0*W0*(A(37)*C20
- A(38)*S20) + W0*(A(39)*S0 + A(40)*C0)
+ (W0**2)*T*(A(39)*C0 - A(40)*S0)
+ W2*(A(41)*C2 - A(42)*S2)

((2. COMPUTE THREE ORBIT ANGLES, THEIR TIME))
((DERIVATIVES AND THE MAGNITUDE OF THE SPACECRAFT))
((POSITION FROM THE FOUR KAMEL PARAMETERS AND))
((THEIR TIME DERIVATIVES:))
(( ))

(( ))
((2.1 COMPUTE R, THE MAGNITUDE OF THE SPACECRAFT))
((POSITION:))
(( ))

R = R0 + DR

(( ))
((2.2 COMPUTE RDOT, THE TIME DERIVATIVE OF THE))
((MAGNITUDE OF THE SPACECRAFT POSITION:))
(( ))

RDOT = DRDT

(( ))
((2.3 COMPUTE I, THE INCLINATION:))
(( ))

I = arcsine(sqrt(LS**2 + PSIS**2))

```
((                                                              ))
((        2.4   COMPUTE ULAT, THE ARGUMENT OF LATITUDE:         ))
((                                                              ))

              IF absolute-value(PSIS) > 0.0D0 THEN
                    ULAT = ATAN2(LS/PSIS)
                ELSE
                    IF LS > 0.0D0 THEN
                          ULAT = 1.570796D0
                    ELSEIF LS = 0.0D0 THEN
                          ULAT = DLAM + GHA + LAM0
                    ELSE
                          ULAT = 4.712389D0
                      ENDIF
               ENDIF

((                                                              ))
((        2.5   COMPUTE RA, THE RIGHT ASCENSION OF THE          ))
((              ASCENDING NODE:                                 ))
((                                                              ))

              RA = DLAM - ULAT + GHA + LAM0

 ((                                                             ))
((      3.  COMPUTE THE POSITION AND VELOCITY VECTORS FROM      ))
((          THREE ORBIT ANGLES, THEIR TIME DERIVATIVES AND THE  ))
((          MAGNITUDE OF THE SPACECRAFT POSITION:               ))
((                                                              ))
((                                                              ))
((        3.1   COMPUTE INTERMEDIATE SINE AND COSINE VALUES:    ))
((                                                              ))

                SU = sine(ULAT)

                CU = cosine(ULAT)

                SI = sine(I)

                CI = cosine(I)

                SRA = sine(RA)

                CRA = cosine(RA)


((                                                              ))
```

((3.2 COMPUTE RU(I) (I=1,2,3), THE (X,Y,Z) COMPONENTS))
((OF THE UNIT VECTOR POINTING TO THE SPACECRAFT))
((POSITION:))

RU(1) = CU*CRA - SU*SRA*CI

RU(2) = CU*SRA + SU*CRA*CI

RU(3) = LS

(( ))
((3.3 COMPUTE FUDOT(I) (I=1,2,3), THE TIME DERIVATIVE))
((OF THE (X,Y,Z) COMPONENTS OF THE UNIT VECTOR))
((POINTING TO THE SPACECRAFT POSITION:))

SRAU = sine(RA - ULAT)

CRAU = cosine(RA - ULAT)

C2I2 = (cosine(I/2))**2

```
RUDOT(1) =  (LSDT*PSIS - PSISDT*LS)*(SRAU/2.0D0/C2I2)
            + (LSDT*LS + PSISDT*PSIS)*(SU*SRA/CI)
            - DLAMDT*(CU*SRA + SU*CRA*CI)
            - GHADT*(CU*SRA + SU*CRA*CI)

RUDOT(2) =  (PSISDT*LS - LSDT*PSIS)*(CRAU/2.0D0/C2I2)
            - (LSDT*LS + PSISDT*PSIS)*(SU*CRA/CI)
            + DLAMDT*(CU*CRA - SU*SRA*CI)
            + GHADT*(CU*CRA - SU*SRA*CI)

RUDOT(3) =  LSDT
```

(( ))
((3.4 COMPUTE POS(I) (I=1,2,3), THE (X,Y,Z)))
((COMPONENTS OF THE SPACECRAFT POSITION:))
(( ))

POS(l) = R*RU(1)

POS(2) = R*RU(2)

POS(3) = R*RU(3)

(( ))
((3.5 COMPUTE VEL(I) (I=1,2,3), THE (X,Y,Z)))
((COMPONENTS OF THE SPACECRAFT VELOCITY:))
(( ))

VEL(1) = RDOT*RU(1) + R*RUDOT(1)

VEL(2) = RDOT*RU(2) + R*RUDOT(2)

VEL(3) = RDOT*RU(3) + R*RUDOT(3)

(( ))
((4. COMPUTE THE KEPLERIAN ELEMENTS:))
(( ))
(( ))
((4.1 SEMI-MAJOR AXIS, SMA:))
(( ))

VELMAG = sqrt(VEL(1)**2 + VEL(2)**2 + VEL(3)**2)

SMA = R/(2.0D0 - (R*(VELMAG**2)/MU))

(( ))
((4.2 ECCENTRICITY, ECC:))
(( ))

RDOTV = POS(1)*VEL(1) + POS(2)*VEL(2) + POS(3)*VEL(3)

P = ((R*VELMAG)**2 - RDOTV**2)/MU

ECC = sqrt(1 - (P/SMA))

(( ))
((4.3 INCLINATION, XINC (THIS HAS ALREADY BEEN))
((COMPUTED FROM THE KAMEL PARAMETERS LS & PSIS):))
(( ))

XINC = I

(( ))
((4.4 RIGHT ASCENSION OF THE ASCENDING NODE, RANODE))
(((THIS HAS ALREADY BEEN COMPUTED FROM THE))
((KAMEL PARAMETER DLAM):))
(( ))

RANODE = RA

```
((                                                                    ))
((          4.5   TRUE ANOMALY, TA:                                   ))
((                                                                    ))

                IF (P - R) > 0.0D0 THEN
                     TA = ATAN2((sqrt(P/MU)*RDOTV)/(P-R))
             ELSE
                     TA = ULAT
             ENDIF

((                                                                    ))
((          4.6   ARGUMENT OF PERIGEE, ARGPER:                        ))
((                                                                    ))

             ARGPER= ULAT-TA

((                                                                    ))
((          4.7   ECCENTRIC ANOMALY, EA:                              ))
((                                                                    ))

             ARG = sqrt[(1.0D0 - ECC)/(1.0D0 + ECC)]

             EA = 2.0D0*arctangent(arg*tangent(TA/2.0D0))

((                                                                    ))
((                SET THE ECCENTRICITY BETWEEN 0 DEGREES              ))
((                (0 RADIANS) AND 360 DEGREES (2*PI RADIANS):         ))
((                                                                    ))

             TWOPI = 2.0D0*3.1415926535898D0


            IF EA < 0.0D0 THEN
                 EA = EA + TWOPI
            ENDIF

((                                                                    ))
((          4.8   MEAN ANOMALY                                        ))
((                                                                    ))

            MA = EA - ECC*sine(EA)


((                                                                    ))
((    THIS COMPLETES COMPUTATION OF THE KEPLERIAN ELEMENTS.           ))
```